Principles of Echolocation
in Bats and Machines

Note 2

Points, Walls and Corners

Why a moving sonar sees more than a sonar standing still

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1 One Image Is Not Enough

An in-air sonar such as our eRTIS sensor sends out a short broadband call and records the echoes with an array of microphones. After matched filtering and beamforming, it produces an image of the echo energy over range r and bearing θ (positive to the left), for the frontal half-plane $|\theta| \leq 90°$. In all figures, the sonar is drawn as a triad: a dot at its position, its pointing direction x (magenta) and its left y (teal). Every bright spot in that image is an echo, and it is tempting to read every spot as an object at that range and bearing. In this note, we show why that is wrong for most of the things a robot meets indoors, and how the motion of the robot tells them apart. We stay in two dimensions (a top view) and use a minimal echo model, with the sensor at position $\boldsymbol{p}$ and four kinds of reflectors: points, walls, corners and round pillars.

2 A Point: the Simplest Reflector

A point reflector, such as a thin pole, a leaf or the edge of a door frame, scatters the incoming sound in all directions. Wherever the sensor is, it receives an echo from the true position $\boldsymbol{q}$ of the point, at range $r = \|\boldsymbol{q} - \boldsymbol{p}\|$ and bearing $\theta = \angle(\boldsymbol{q} - \boldsymbol{p})$. The sound spreads on the way out and again on the way back, so the echo amplitude falls with the square of the range,

$$A_{\text{point}} = \frac{\sigma}{r^2}, \tag{1}$$

with σ a measure of how strongly the point scatters. The sonar does not see the point as a point. It sees the point-spread function (PSF) of the sensor, centered on (r, θ):

$$I(r', \theta') = A\, h(r' - r,\ \theta' - \theta). \tag{2}$$

In range, the PSF is as narrow as the bandwidth of the call allows (here a few centimeters). In bearing, it is the beam pattern of the array (here a 10 cm aperture with a Hann taper, which keeps the sidelobes below -30 dB). It is narrowest straight ahead and widens towards $\pm 90°$, where the array sees its own aperture edge-on. The PSF is therefore anisotropic: a few centimeters in range everywhere, but in bearing 6° wide straight ahead, 8° at 45° and 18° at 70° (Fig. 1c). For a line aperture, the beam has a fixed width in $\sin\theta$, and $\sin\theta$ changes ever more slowly with θ towards the side, so the width in bearing grows as $1/\cos\theta$, until the edge of the field of view cuts the beam off. Figure 1 shows three points and their image.

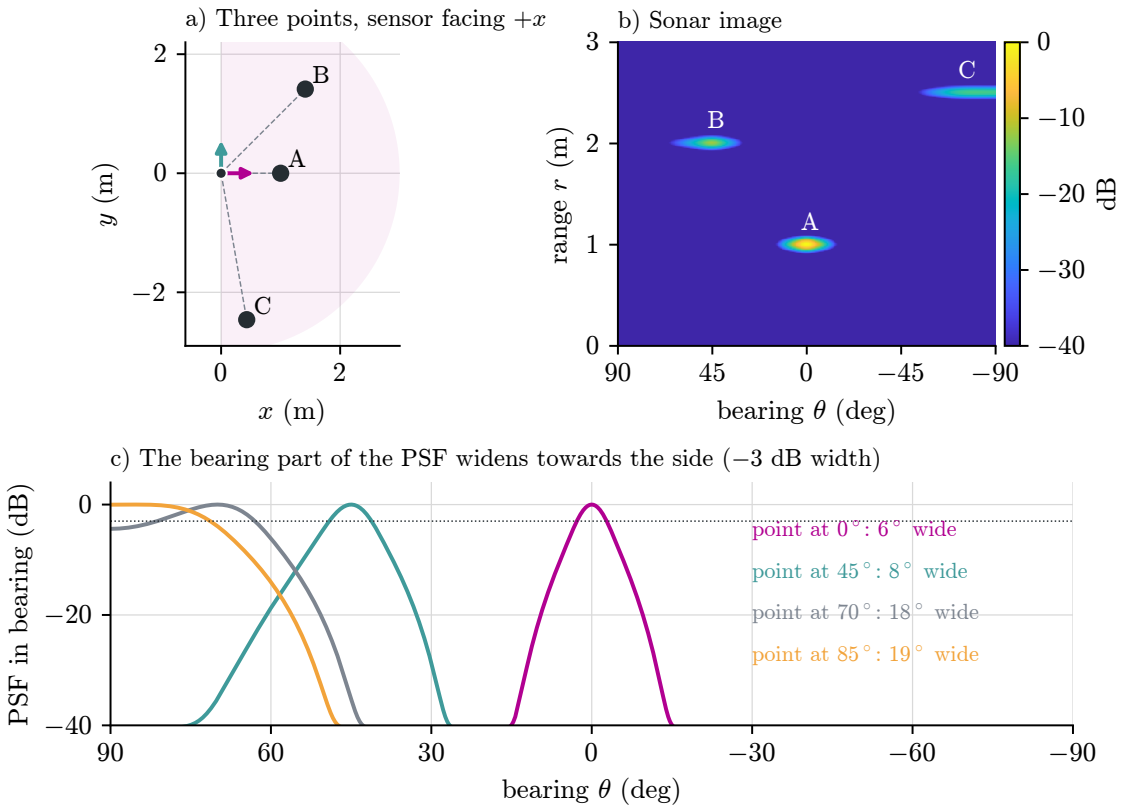


Figure 1: Three points and the sonar image they produce. Every point becomes a copy of the point-spread function, weaker with range, and wider in bearing towards the side (C, at −80°). c) The bearing part of the PSF for points at four bearings: the further to the side, the wider the PSF.

3 A Wall: a Point That Moves With You

A smooth wall is a mirror for sound, because its roughness is much smaller than the wavelength (about 7 mm at 50 kHz). A mirror does not scatter in all directions. It reflects every ray at an equal angle, and the only ray that comes back to the sensor is the one that hits the wall perpendicularly. The echo therefore comes from the *foot of the perpendicular* from the sensor to the wall, at a range d equal to the distance to the wall (Fig. 2a). Seen from the sensor, the echo looks like the echo of a mirror image of the sensor itself, the image source, at distance $2d$ behind the wall. Because the sound spreads only once over the total path $2d$, the echo is strong:

$$A_{\text{wall}} = \frac{R}{2d}, \tag{3}$$

with R the reflection coefficient of the wall (close to 1). At 1 m, a wall is about 20 dB stronger than a point with $\sigma = 5$ cm.

In a single image, the wall echo is indistinguishable from a point at the foot of the perpendicular. The difference appears as soon as the robot moves. When the robot drives along the wall, the foot of the perpendicular moves with it. The echo stays at the same range and the same bearing, 90° (Fig. 2b,c). A true point at the same place behaves completely differently: it comes closer, sweeps through the bearings and leaves the field of view. The wall echo looks like a point, but a point that never comes closer, because every measurement sees a different part of the wall.

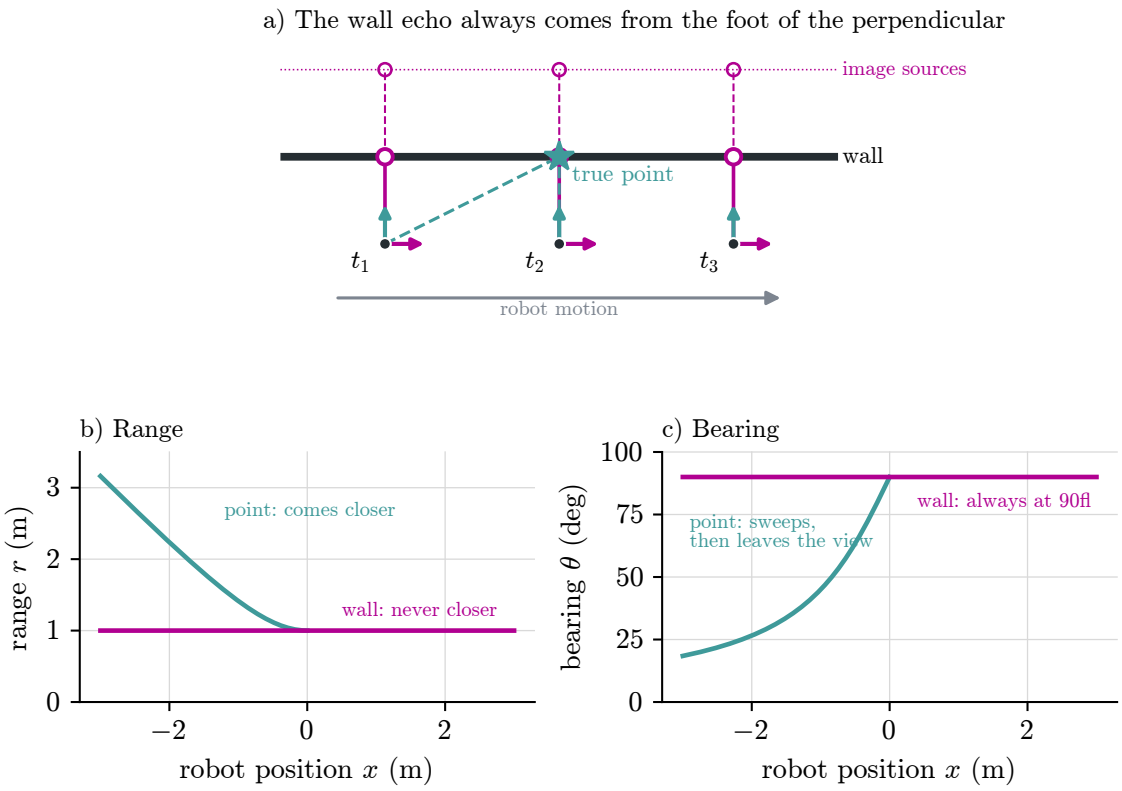


Figure 2: A wall against a true point, for a robot driving along the wall. a) At every position t_1, t_2, t_3, the wall echo comes from the foot of the perpendicular (open circles), the mirror image of the sensor sits behind the wall. The true point (star) is seen from t_1 and t_2, and has left the field of view at t_3. b, c) Range and bearing of the two echoes along the drive.

Key idea 1

A specular wall returns a single echo from the foot of the perpendicular. It looks like a point, but under motion it does not behave like one: it travels with the robot, at constant range and bearing.

4 A Corner: the Strongest Point in the Room

Where two walls meet at a right angle, an inside corner forms, and corners are remarkable reflectors. A ray that enters the corner bounces off both walls and leaves in exactly the opposite direction, whatever its angle of incidence (Fig. 3a). Each reflection mirrors the direction of the ray in one axis, and two perpendicular mirrors together reverse it completely. In terms of image sources, the double reflection mirrors the sensor at $\boldsymbol{p}$ through the apex $\boldsymbol{c}$ of the corner, to $2\boldsymbol{c}-\boldsymbol{p}$. The line from the sensor to this image source passes through the apex, so the echo appears exactly at the corner, at range $r = \|\boldsymbol{c}-\boldsymbol{p}\|$, with

$$A_{\text{corner}} = \frac{R^2}{2r}. \tag{4}$$

A corner thus combines the best of both reflectors. Like a wall, it is a mirror and therefore strong: at 2.5 m, it is almost 30 dB stronger than a point. Like a point, it sits at a fixed place in the world, so it comes closer and sweeps through the bearings as the robot drives past. Moreover, it returns the sound from every direction within its 90° opening, whereas a flat panel only answers when the sensor looks at it perpendicularly. Because it sends the sound back to

where it came from, a corner is called a retroreflector, and this is why corner reflectors are used as targets to calibrate radars and sonars, and why the corners of a room are among the most reliable landmarks for a sonar-guided robot.

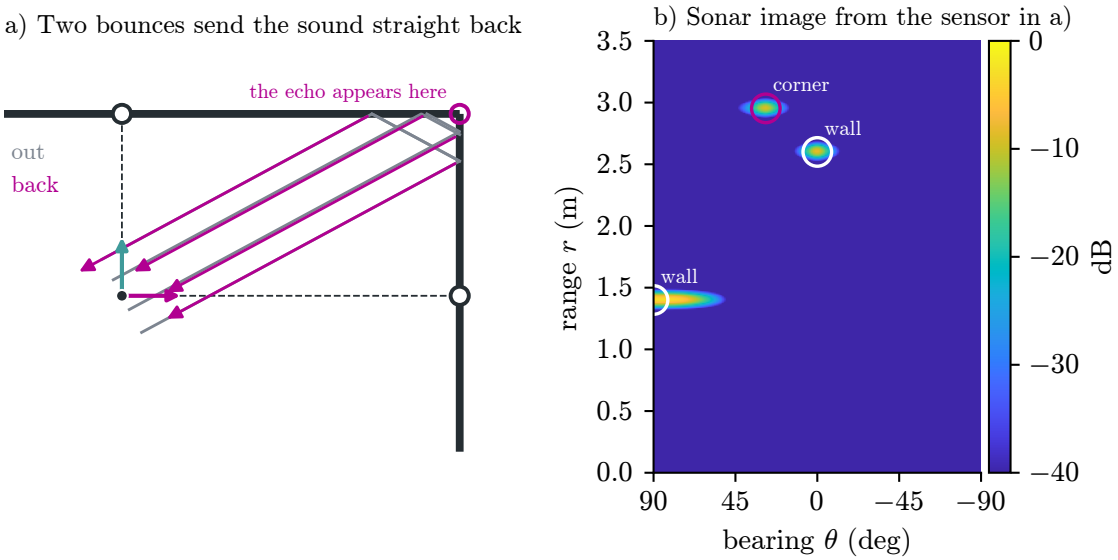


Figure 3: A corner. a) Every ray that enters the corner bounces twice and returns parallel to itself, so the corner echo appears at the apex (magenta circle). The open circles are the reflection points of the two walls themselves, at the feet of their perpendiculars. b) The sonar image from the sensor in a), with the same echoes circled.

Key idea 2

A corner reflects like a mirror but moves like a point. It sends the sound back where it came from (a retroreflector), so it is strong, visible from a wide range of directions, and fixed in the world: an ideal landmark.

5 Driving Through Four Corridors

We can now predict what a robot sees when it drives straight through a corridor 2 m wide. Figure 5 shows four corridors, each with three poses of the robot only 35 to 40 cm apart, and the sonar image at each pose. Compare the three images of a row: what moves, and what stays where it is?

Points (a). Every echo moves between the images. Points ahead come closer and drift outward, towards ±90°, where they leave the field of view, while new points appear far ahead. Together, they form the characteristic flow of a static world past a moving sensor, the acoustic flow.

Smooth walls (b). There are only two echoes, one on each side, at ±90° and 1 m, and the three images are identical. Each echo comes from a different part of the wall, the part next to the robot, but nothing in the images moves. A tracker that assumes static points would conclude that two objects drive along with the robot.

Walls with strong points (c). Real walls are rarely smooth everywhere. Door frames, pipes, fire extinguishers and signs stick out and act as strong points. The images now combine both behaviors: the two wall echoes stay at ±90° and 1 m, while the points on the walls come closer and drift outward, exactly like the points in (a). Only the moving echoes tell the robot where along the corridor it is.

Round pillars (d). A cylinder is a curved mirror. Its echo comes from the point on its surface that faces the sensor, a glint, in the direction of its center and at range $\|\boldsymbol{c} - \boldsymbol{p}\| - a$ for a radius a. The curvature spreads the reflected sound, so the glint is weaker than a wall but much stronger than a point,

$$A_{\text{cyl}} = \frac{1}{2r}\sqrt{\frac{a}{r+a}}. \tag{5}$$

In the images, pillars move almost like points, but strongly, and with a range that never drops below the distance to their surface. Their echo does not come from a fixed place, however: as the robot passes, the glint slides around the pillar, over the half that faces the corridor.

Figure 4 zooms in on one pillar. From every pose, the glint sits where the surface faces the sensor, so it slides around the pillar as the robot drives past (b). Compared with a fixed point on the front of the pillar, the glint is up to 15 cm closer and up to 6° further to the side while the pillar is still ahead, and the difference vanishes when the robot is abeam (c). The pillar echo thus comes closer like a point, but along a slightly different path: it wiggles with respect to every fixed point, and a tracker that assumes a point makes small, systematic errors. The larger the pillar, the larger the wiggle, and for $a \to 0$ the pillar becomes a point.

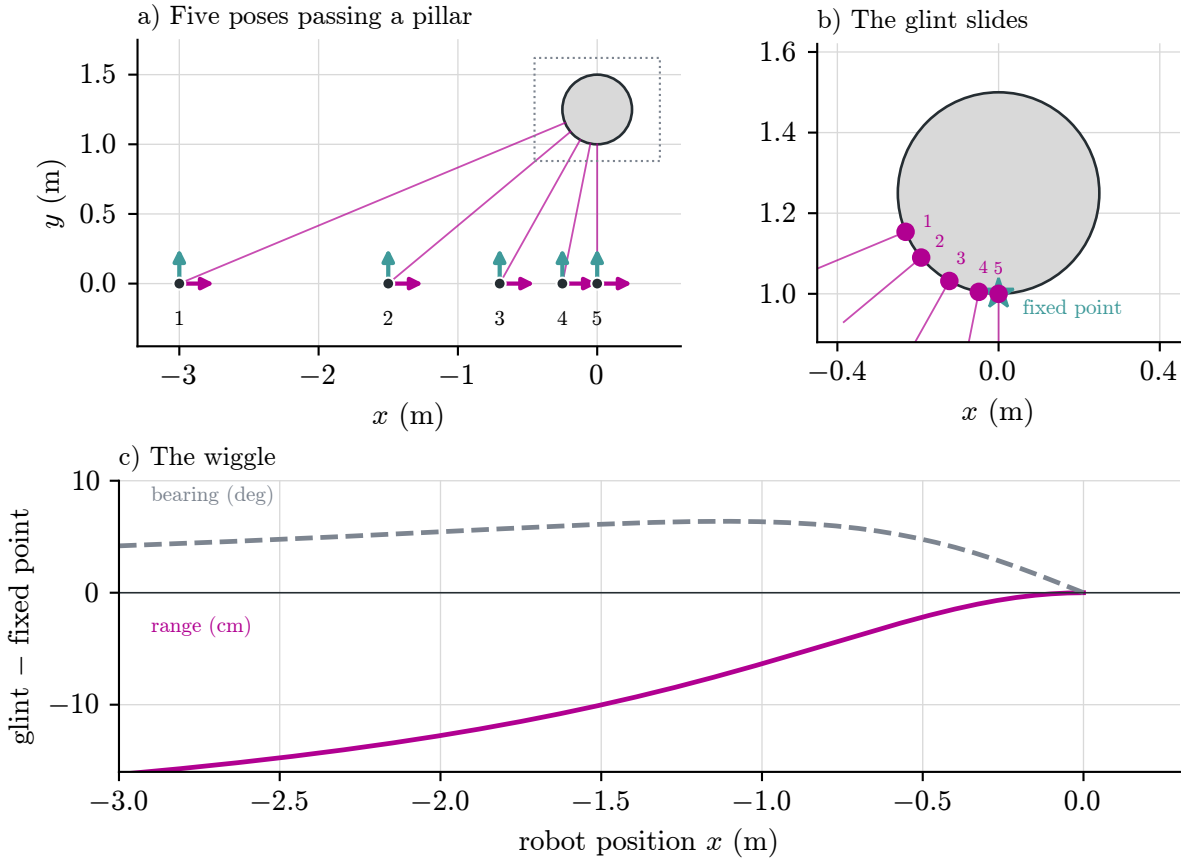


Figure 4: The glint of a round pillar (radius 25 cm) moves. a) Five poses of the robot passing the pillar, with the line of sight to the glint. b) Zoom on the pillar (dotted box in a): the glint seen from each pose (numbered), and a fixed point on the front of the pillar (star). c) Range and bearing of the glint minus those of the fixed point.

6 Why Motion Matters

Table 1 summarizes the five reflectors. In a single sonar image, they all look the same: a bright spot with the shape of the point-spread function. Their amplitudes differ, but amplitude alone

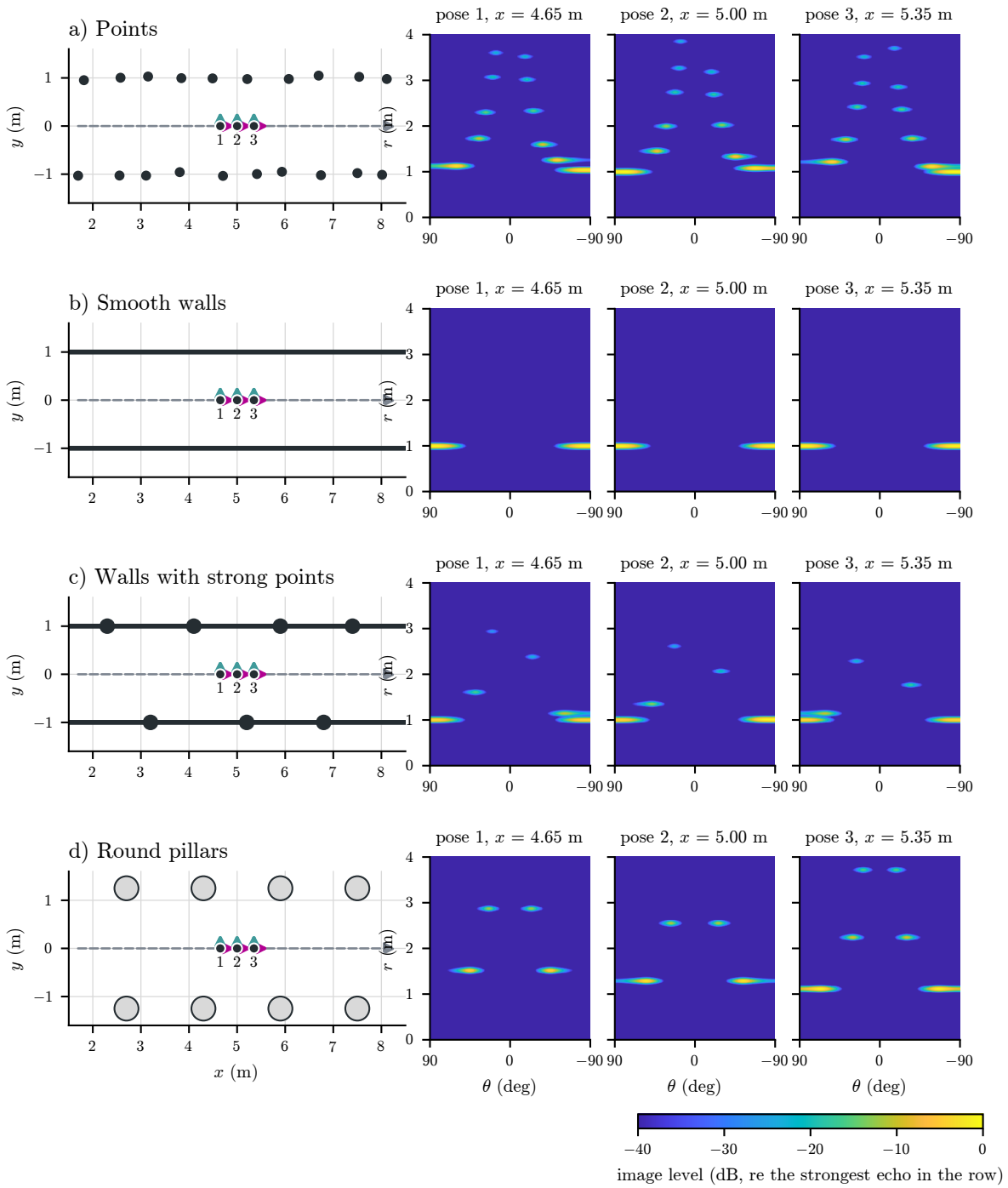


Figure 5: A robot driving through four corridors. Left: the geometry and three poses of the sonar (triads 1 to 3). Right: the sonar image at each pose, in dB relative to the strongest echo in the row. Points (a) and pillars (d) move between the images, the smooth walls (b) do not, and in (c) the strong points on the walls move while the walls stay put.

Reflector	Amplitude	Echo appears at	Under motion	In the world
Point	σ / r^2	the point	comes closer, sweeps	fixed
Wall	$R / 2d$	foot of the perpendicular	constant r and θ	slides along the wall
Corner	$R^2 / 2r$	the apex	like a point	fixed
Edge, fixture	σ / r^2	the edge	like a point	fixed
Pillar	$\frac{1}{2r}\sqrt{\frac{a}{r+a}}$	glint facing the sensor	almost like a point	slides around the pillar

Table 1: The five reflectors of this note.

is ambiguous, because a weak echo can be a strong reflector far away or a weak one nearby. What separates them is how their echoes move when the robot moves. Only points, corners and edges are fixed in the world. Walls and pillars produce echoes whose apparent position moves with the sensor. A robot that builds a map from its sonar, or a bat that flies through a forest, should therefore never interpret a single echo on its own. The motion of the sensor, and the way the echoes follow or do not follow it, is part of the measurement.

Key idea 3

Points, edges and corners are fixed in the world, walls and curved surfaces are not. One image cannot tell them apart. The echoes of a moving sensor can.

Exercise

1. A robot drives towards a wall, perpendicular to it. How do range and bearing of the wall echo change? Can the robot now tell the wall from a point?

2. A corridor ends in a T-junction. Which new echoes appear in the image as the robot approaches the junction, and how do they move?

3. The pillar glint slides around the pillar. For a robot passing at distance D from the pillar's center, over which angle does the glint travel while the pillar is in the field of view? What happens for $a \to 0$?

Further Reading

- R. Kuc and M. W. Siegel, "Physically based simulation model for acoustic sensor robot navigation," IEEE Trans. Pattern Anal. Mach. Intell., 9(6), 766–778, 1987.

- J. J. Leonard and H. F. Durrant-Whyte, *Directed Sonar Sensing for Mobile Robot Navigation*, Kluwer, 1992.

- J. Steckel and H. Peremans, "BatSLAM: Simultaneous localization and mapping using biomimetic sonar," PLoS ONE, 8(1), e54076, 2013.