Principles of Echolocation
in Bats and Machines

Note 8

Fluttering Wings, Fluttering Echoes

How a beating wing writes itself into an echo

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Version 2026-11-18 16:28

1 A Moth Is a Flapping Mirror

In the note on points, walls and corners, we saw that a smooth wall is a mirror for ultrasound: it only sends an echo back when the sonar looks at it perpendicularly. The wing of an insect is, acoustically, a very small wall. It is thin, smooth and a few centimeters long, which is several wavelengths at the frequencies bats and our sonars use (about 4 mm at 80 kHz). But unlike a wall, the wing does not stay put. It beats up and down, from about ten times per second for large moths to several hundred times per second for small flies and mosquitoes, and every time it turns to face the sonar, it flashes. These flashes are called acoustic glints. Between them, mostly the much weaker echo of the body remains.

Glints matter twice. For an echolocating bat, they are the signature of a flying insect, a pattern that no leaf or branch produces, and some bats are specialized in hearing it. For a robot sonar, they turn an insect, or any moving mechanism such as a fan, into a target whose echo changes from one call to the next, which is easy to misread. In this note, we build a minimal model of a fluttering insect and look at its echo through two kinds of calls: a long constant-frequency (CF) call, as horseshoe bats use, and a short frequency-modulated (FM) sweep, as most other bats and our eRTIS sensor use. We then look at both through the joint acoustic and modulation frequency (JAMF) transform, which sorts echoes by how fast they fluctuate.

2 A Minimal Model

We look at the insect from above (Fig. 1). The sonar sits at the origin and points along x, drawn as a triad (magenta: the pointing direction, teal: its left). The insect hovers at a range of $R_0 = 1$ m. Its body is a weak point reflector, -20 dB below a full glint. A single wing, a flat strip of 2 cm, is hinged at the body and flaps around it,

$$\beta(t) = \beta_0 \sin(2\pi f_w t) + \beta_{\text{off}}, \tag{1}$$

with β the angle between the wing and the direction perpendicular to the line of sight, a flapping amplitude $\beta_0 = 60°$, a slight asymmetry β_{off} and $f_w = 47$ wingbeats per second. We split the wing into closely spaced point reflectors and add up their echoes, each with its own travel time at the moment of reflection. Nothing else goes into the model: the glints and the Doppler shifts that we will see follow from the geometry alone.

The echo of a flat strip of length L is strong only when the path lengths to all of its points are equal to within about half a wavelength, that is, when

$$|\sin\beta| < \frac{\lambda}{2L}. \tag{2}$$

At 80 kHz and $L = 2$ cm, this is a window of only $\pm 6°$ around the moment the wing faces the sonar. Since the wing sweeps through that window quickly, twice per wingbeat (once on the downstroke and once on the upstroke), each glint is short.

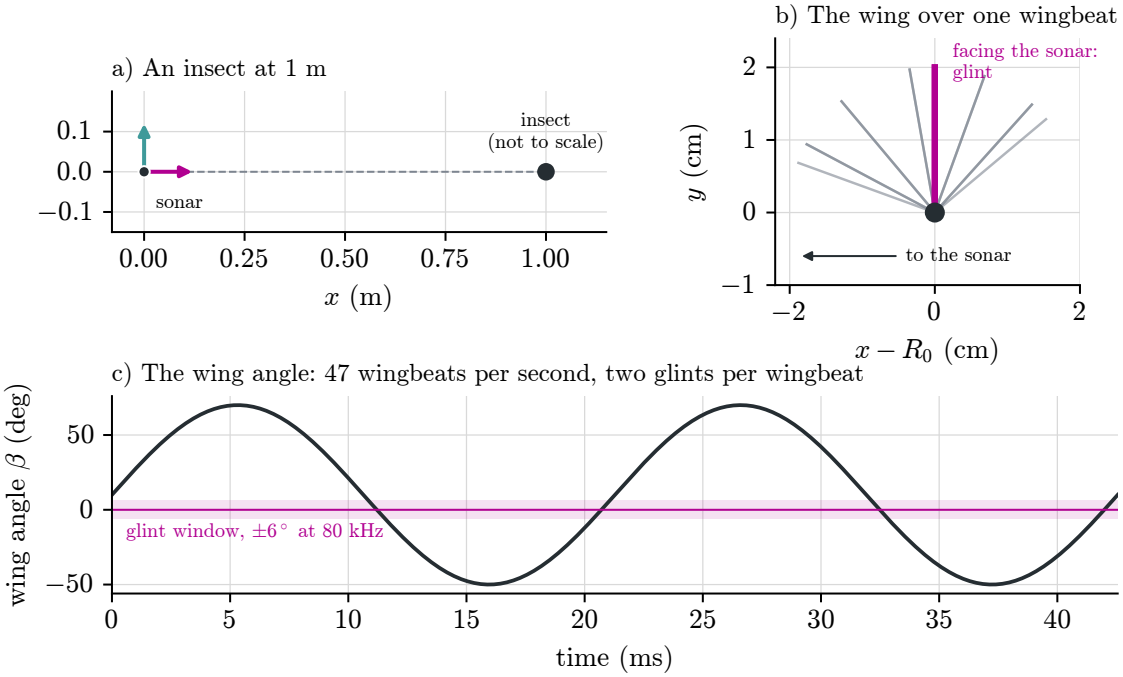


Figure 1: The model. a) A sonar (triad) and an insect at 1 m. b) The wing over one wingbeat (gray), and the orientation in which it faces the sonar (magenta). c) The wing angle over two wingbeats: the wing passes through the narrow glint window (shaded) twice per wingbeat.

3 How Short Is a Glint, and How Wide Is Its Doppler?

Two numbers characterize a glint: how long it lasts and how far it spreads in frequency. Both follow from the motion of the wing. The wing angle changes fastest in the middle of a stroke, at a rate $\beta_0 \cdot 2\pi f_w$, which is also roughly where the wing faces the sonar. A point at distance x from the hinge then moves along the line of sight at $v(x) = x\,\beta_0\,2\pi f_w$, and the tip, at $x = L$, at $v_{\text{tip}} = L\,\beta_0\,2\pi f_w$. A reflector moving at speed v towards the sonar shifts the echo by $2vf_0/c$, so the echo of the wing spreads over Doppler shifts from zero (at the hinge) up to $\Delta f_{\max} = 2v_{\text{tip}}f_0/c$. The glint, in turn, lasts as long as the wing needs to sweep through the window $|\beta| < \lambda/(2L)$, which has a total width of about λ/L (for small angles). At the angular rate $\beta_0\,2\pi f_w$, that takes

$$\tau \approx \frac{\lambda/L}{\beta_0\,2\pi f_w} = \frac{c}{f_0\,L\,\beta_0\,2\pi f_w}. \tag{3}$$

For our moth-like wing ($L = 2$ cm, $\beta_0 = 60°$, $f_w = 47$ Hz) at 80 kHz, the tip moves at 6.2 m/s, the Doppler spread reaches 2.9 kHz, and the glint lasts about 0.7 ms (the simulated glint is 0.4 ms wide at half its amplitude). Multiplying $\Delta f_{\max}$ and τ gives a remarkably simple result,

$$\Delta f_{\max} \cdot \tau \approx 2, \tag{4}$$

independent of the size, the speed and the frequency. A glint is a short event, and a short event must be wide in frequency: a faster or larger wing makes the glint shorter and its Doppler spread wider in exactly the same proportion.

4 A Long Constant-Frequency Call

Figure 2a shows the echo envelope of a 50 ms call at 80 kHz, about the call length and frequency of a horseshoe bat. Most of the time, the echo sits at the level of the body. Four times during the call, twice per wingbeat, a glint appears, about 21 dB above the body echo and only about 0.4 ms wide at half amplitude. The insect announces itself in short, regular flashes.

The spectrogram (Fig. 2b) shows a second signature. A moving reflector shifts the frequency of its echo by $\Delta f = 2v f_0/c$, with v its speed towards the sonar. The body hovers, so its echo stays at 80 kHz. The wing, however, rotates, and at the moment of a glint its tip moves towards or away from the sonar at up to 6.2 m/s. Each glint therefore smears the echo over a band of Doppler shifts of up to ±2.9 kHz, and between the glints, weaker echoes from the edges of the wing trace the sinusoidal velocity of the wing. In radar, the same effect of rotating or vibrating parts (helicopter blades, the swinging arms of a walking person) is called micro-Doppler.

Both signatures follow from one expression. Let the call be $s(t) = \cos(2\pi f_0 t)$, and let element i of the wing be at range $r_i(t)$. Its echo arrives $2r_i(t)/c$ later, so the received signal is the sum

$$e(t) = \sum_i a_i \cos\Big(2\pi f_0\big(t - 2r_i(t)/c\big)\Big) = \mathrm{Re}\big\{A(t)\, e^{j2\pi f_0 t}\big\}, \qquad A(t) = \sum_i a_i\, e^{-j4\pi f_0 r_i(t)/c}, \quad (5)$$

with the body as one more term of constant range. All the motion of the insect sits in the complex envelope $A(t)$. Its magnitude is the envelope of Fig. 2a: it peaks when the phases of all elements line up, which is a glint. The rate of change of its phase, divided by 2π, is the Doppler shift that Fig. 2b follows. The wing repeats itself every wingbeat, so $A(t)$ is periodic with period $1/f_w$ and can be written as a Fourier series, $A(t) = \sum_n A_n e^{j2\pi n f_w t}$. The echo spectrum is then a set of lines at $f_0 + n f_w$, sidebands spaced by the wingbeat rate and reaching out to about $\pm\Delta f_{\max}$.

The spectrogram does not resolve these lines. It is the squared magnitude of the short-time Fourier transform (STFT),

$$X(t, f) = \int e(t')\, w(t' - t)\, e^{-j2\pi f t'}\, dt', \quad (6)$$

which looks at the echo through a window w of length T_w (here 1 ms) and has a frequency resolution of about $1/T_w = 1$ kHz, far coarser than the 47 Hz line spacing. A window longer than a wingbeat would separate the lines, but would blur the glints. What the spectrogram does show is that its power $P(t, f) = |X(t, f)|^2$ repeats itself in time, with the period of the wingbeat, in every frequency bin. We return to this periodicity in Section 7.

This is why long CF calls suit flutter detection so well. The call catches several glints, and against the steady echo of the background (leaves and branches do not flap), the periodic flashes in amplitude and frequency stand out. Horseshoe bats, whose calls are dominated by such a long CF component, are the classic example. Their hearing is extremely sharply tuned to a narrow band around the frequency of the returning echo (an acoustic fovea), and they lower the frequency of their call while flying, so that the Doppler shift of their own flight brings the echo back into that band (Doppler shift compensation). Flutter then shows up as modulations around a fixed, finely resolved frequency [1, 2].

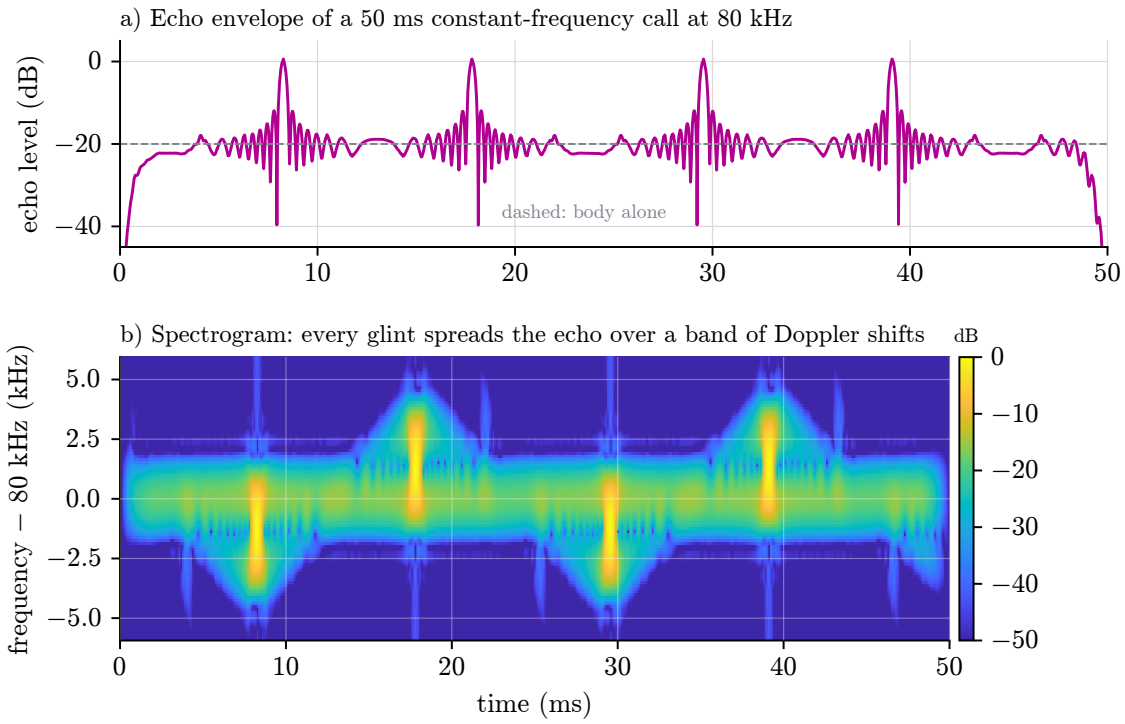


Figure 2: The echo of a long constant-frequency call (50 ms at 80 kHz) from the fluttering insect. a) Envelope: short glints, twice per wingbeat, far above the echo of the body. b) Spectrogram (1 ms windows): every glint spreads the echo over a band of Doppler shifts, and the weaker echoes in between follow the velocity of the wing. Glints of this kind, and the information they carry for a bat, were measured on real insects by Kober and Schnitzler [3].

Key idea 1

A flapping wing is a small mirror that faces the sonar twice per wingbeat. Its echo is a train of short glints, each spread in frequency by the motion of the wing (micro-Doppler). Long constant-frequency calls catch several glints and show them as modulations of an otherwise steady echo.

5 A Moth and a Mosquito

Not every insect glints. The glint needs a wing that is several wavelengths long, so that its echo is directional. For a mosquito, with wings of about 2.5 mm, about 500 wingbeats per second and an unusually small stroke amplitude (here $\pm 20°$), the numbers come out very differently (Table 1). Its wingtip is slower than the moth's, despite the ten times higher wingbeat rate, because the wing is so short. At 80 kHz its wing is shorter than one wavelength, so the glint window, $\sin\beta < \lambda/(2L)$, covers almost every angle the wing takes. The wing reflects about equally in every position, and instead of short flashes, the echo shows a gentle modulation of only 4 dB at twice the wingbeat rate, against more than 40 dB for the moth. It is also much weaker: about -18 dB below the moth's glint in our two-dimensional model, and even more in three dimensions, where the echo scales with the area of the wing rather than its length.

	moth (model)	mosquito (model)
wing length	20 mm	2.5 mm
wingbeat rate	47 Hz	500 Hz
stroke amplitude	±60°	±20°
tip speed	6.2 m/s	2.7 m/s
maximum Doppler shift at 80 kHz	2.9 kHz	1.3 kHz
glint window at 80 kHz	±6°	±59° (wider than the stroke)
glint duration	0.7 ms	no glint
wing echo, peak (re moth glint)	0 dB	-18 dB
wing echo, peak to trough	>40 dB	4 dB

Table 1: Two model insects at 80 kHz. The numbers follow from the equations in the text and, for the wing echo, from the simulation of a 30 ms CF call. The mosquito parameters are rough, typical values.

6 Short FM Calls

A short FM sweep is a different instrument. The eRTIS call, a 2.5 ms sweep from 75 to 20 kHz, is short compared with a wingbeat (47 wingbeats per second means one every 21 ms), so each call takes a snapshot of the wing. Whether that snapshot contains a glint depends on where in its stroke the wing happens to be. Over one wingbeat, the matched-filter output of our model varies between -25 and -6 dB relative to a full glint (Fig. 3, gray): a call that overlaps a glint returns a strong echo, while a call between two glints mostly hears the body. (The FM echo never reaches the full glint, because the glint lasts only a fraction of the call.)

A sonar that calls at a fixed rate samples this pattern, and sampling a fast periodic process slowly causes aliasing. At 100 calls per second, a rate that a bat reaches in its final approach to prey, the echo level still flickers from call to call, but the glints are caught only now and then, and their pattern beats slowly against the call rate (Fig. 3a). At 10 calls per second, a typical rate for a robot sonar, almost every call misses the glints, and the few that hit them line up into a slow, false modulation that has nothing to do with the real wingbeat (Fig. 3b). It is the wagon wheel that seems to turn backwards in an old western.

We can make this quantitative. The echo level is a periodic function of time with the period of the wingbeat. A sonar that calls f_c times per second samples it, and a periodicity at f_w then appears at the apparent rate

$$f_a = |f_w - k f_c|\,, \qquad k = \mathrm{round}\left(\frac{f_w}{f_c}\right), \tag{7}$$

which always lies between 0 and $f_c/2$ (Fig. 3d). Our moth, at 47 Hz, appears at 3 Hz to a sonar calling 10 times per second, and at 47 Hz (that is, unaliased) only for call rates above $2f_w \approx 94$ per second. A mosquito would need more than 1000 calls per second. Following the glints themselves, which last a fraction of a millisecond, would take thousands of calls per second, far beyond any sonar, since the next call can only start once the echoes of the previous one have died out. A sonar with short calls therefore always sees snapshots. What fraction of those snapshots contains a glint? A call of duration T overlaps one of the two glints per wingbeat whenever it starts within a window of about $T + \tau$ before it, so the fraction is about

$$p \approx 2 f_w\,(T + \tau), \tag{8}$$

30% for our moth and the 2.5 ms eRTIS call. Counting the calls in our simulation whose echo rises more than 5 dB above the body echo gives 30%.

The spectrum of each echo tells more than its level. During a 2.5 ms call the wing turns by tens of degrees, far more than the glint window, so the call does not see one frozen wing. But a sweep is at each frequency only for a moment: our hyperbolic sweep reaches frequency f at the

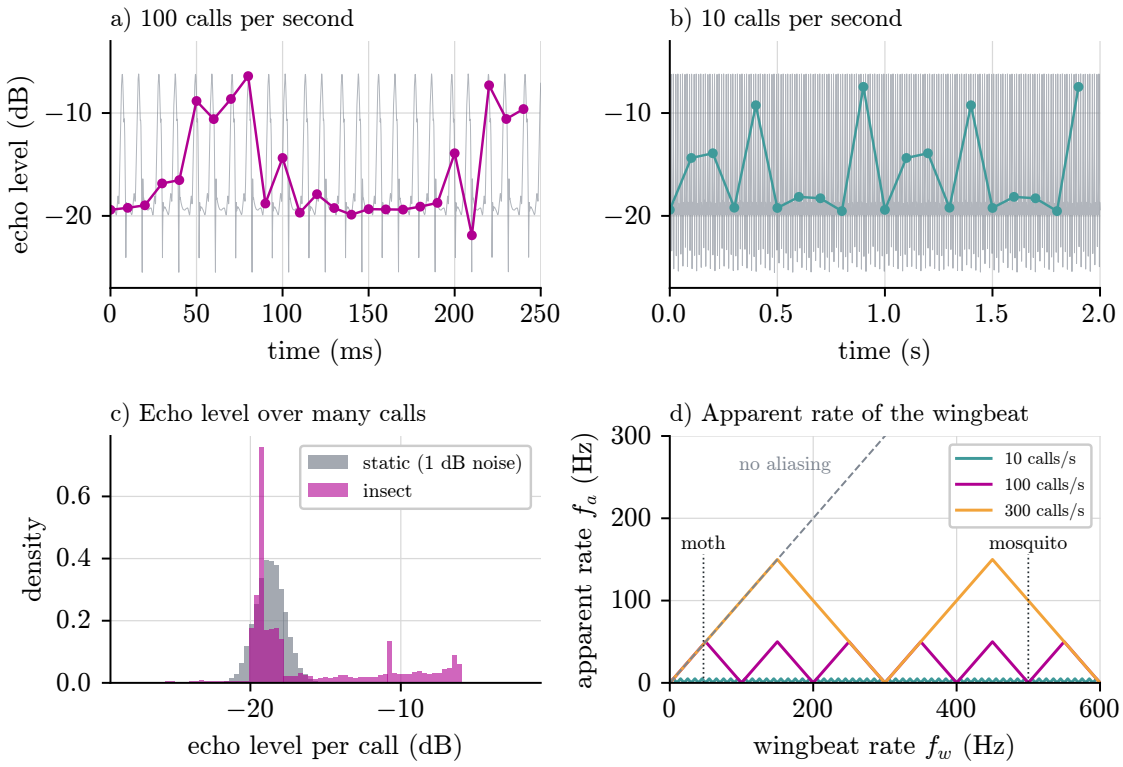


Figure 3: The echo level of a short FM call (2.5 ms, 75 to 20 kHz, matched filter) from the fluttering insect, in dB relative to a full glint. Gray: the level for a call emitted at any moment, colored dots: the calls of a sonar calling a) 100 and b) 10 times per second. c) The distribution of the echo level over 20,000 calls at random moments, for the insect and for a static reflector with 1 dB of measurement noise. d) The apparent rate of the echo modulation against the true wingbeat rate, for three call rates. Only on the dashed line (call rate above twice the wingbeat rate) does the sonar see the true wingbeat.

time $t_s(f) = (1/f - 1/f_{\text{start}})/b$ after its start, with b set by its duration and end frequencies. Each frequency therefore sees the wing as it was at its own moment, and the echo of call m, emitted at t_m, is approximately the call filtered by the insect at that moment,

$$E_m(f) \approx S(f)\, H\big(f,\, t_m + t_s(f)\big), \qquad H(f,t) = \sum_i a_i\, e^{-j4\pi f r_i(t)/c}, \tag{9}$$

with $S(f)$ the spectrum of the call and H the same sum as $A(t)$ in Eq. (5), now evaluated at every frequency of the sweep. A glint lights up only the frequencies the sweep passes while the wing faces the sonar, and which ones depends on when the call was emitted. Figure 4a shows $|E_m(f)|^2/|S(f)|^2$ for a call emitted at any moment: the glints are curved stripes, 1.8 ms earlier at 25 kHz than at 70 kHz, exactly as Eq. (9) predicts (dashed lines). A sonar that calls 100 times per second sees only samples of this pattern, and its echo spectrum changes from ping to ping (Fig. 4b): bright at high frequencies in one ping, at low frequencies in another, and mostly dim. This sequence of spectra, against the slow time of the pings, is the spectrogram of a pulsed sonar. Since H repeats every wingbeat, its power is again periodic in every frequency bin.

Key idea 2

A short FM call takes a snapshot of the wing. A sonar calling at a fixed rate samples the glints, and unless it calls much faster than the wingbeat, it sees a flicker, or even a slow false modulation, instead of the wingbeat itself.

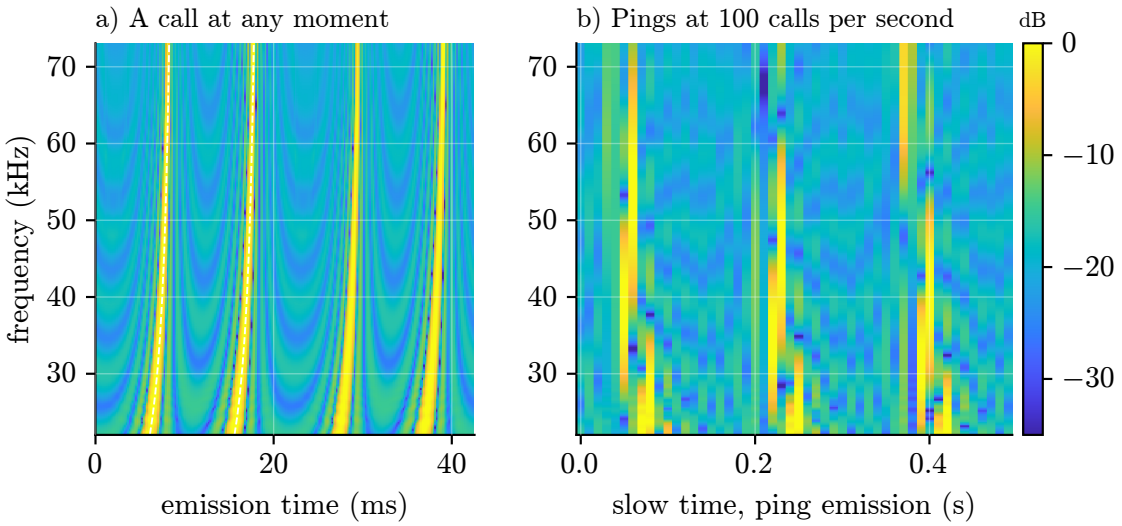


Figure 4: The echo spectrum of short FM calls (2.5 ms, 75 to 20 kHz) from the fluttering insect, divided by the spectrum of the call, in dB relative to a full glint. a) For a call emitted at any moment over two wingbeats. Each glint is a curved stripe, because the sweep reaches the low frequencies later. Dashed: Eq. (9) for the glints of the first wingbeat. b) The pings of a sonar calling 100 times per second, against slow time.

7 The Joint Acoustic and Modulation Frequency Transform

The spectrogram of the CF echo and the ping-by-ping spectra of the FM echoes share one property: in every acoustic frequency bin, the power rises and falls with the wingbeat. The joint acoustic and modulation frequency (JAMF) transform measures exactly this [5, 6]. It takes the power of the STFT, $P(t, f) = |X(t, f)|^2$, and Fourier-transforms every acoustic frequency bin over time,

$$J(f, \alpha) = \left| \int P(t, f)\, e^{-j2\pi\alpha t}\, dt \right| . \tag{10}$$

The new variable α is the modulation frequency, how fast the power in the band around f fluctuates, and the result is a map of acoustic frequency against modulation frequency. (Some authors use $|X|$ instead of the power, which changes nothing below.)

A static reflector, a leaf or a branch, returns the same power at every moment, so its $P(t, f)$ does not depend on t, and all of its JAMF sits at $\alpha = 0$. The wing makes $P(t, f)$ periodic with period $1/f_w$, so in every bin it is a Fourier series, $P(t, f) = \sum_k p_k(f)\, e^{j2\pi k f_w t}$, and the JAMF has lines at $\alpha = k f_w$ with strengths $|p_k(f)|$, at the acoustic frequencies where the glints live. Because a glint lasts only a small fraction of a wingbeat, many harmonics are strong. Where a static echo overlaps the insect in frequency, it adds cross terms to P, but these repeat with the wing too and land on the same lines. Energy at $\alpha \neq 0$ therefore means that something moves, however strong the clutter at $\alpha = 0$. Figure 5a and b show this for the CF echo of our moth in front of a static leaf 6 dB below a full glint. The leaf and the body form the bright column at zero modulation frequency, and the wing adds lines at multiples of 47 Hz across the band of Doppler shifts, the strongest 24 dB below that column. The resolution on the modulation axis is about one over the observation time. The 44 ms steady part of a 50 ms call gives 23 Hz, so the lines barely separate (a), while a 200 ms call gives sharp lines (b). The 1 ms STFT window sets the other end and follows modulations up to about 1 kHz. Reading the wingbeat rate from a single call thus takes a call that lasts several wingbeats, one more reason why long CF calls suit flutter.

A pulsed FM sonar knows $P(t, f)$ only at its pings, $P_m(f) = |E_m(f)|^2$ at $t_m = m/f_c$ for a call rate f_c, and the integral becomes a sum over slow time, $J(f, \alpha) = \left| \sum_m P_m(f)\, e^{-j2\pi\alpha m/f_c} \right|$.

This sum repeats itself in α with period f_c, so the modulation axis ends at $f_c/2$, the modulation Nyquist frequency, and every wingbeat harmonic kf_w folds back to $|kf_w - nf_c|$, the apparent rate of Eq. (7). At 100 calls per second, the axis ends at 50 Hz, and the first four harmonics of our moth land at 47, 6, 41, 12 Hz (Fig. 5c). At 10 calls per second, it ends at 5 Hz, the same harmonics land at 3, 4, 1, 2 Hz, and every tenth harmonic falls on $\alpha = 0$, where the clutter lives (Fig. 5d). The insect still has energy away from zero, so the JAMF still tells it from the leaf, but it no longer tells the wingbeat rate. In our lab, the JAMF first served a very different signal: lung sounds, where it gave a compact representation for computational analysis [7, 8].

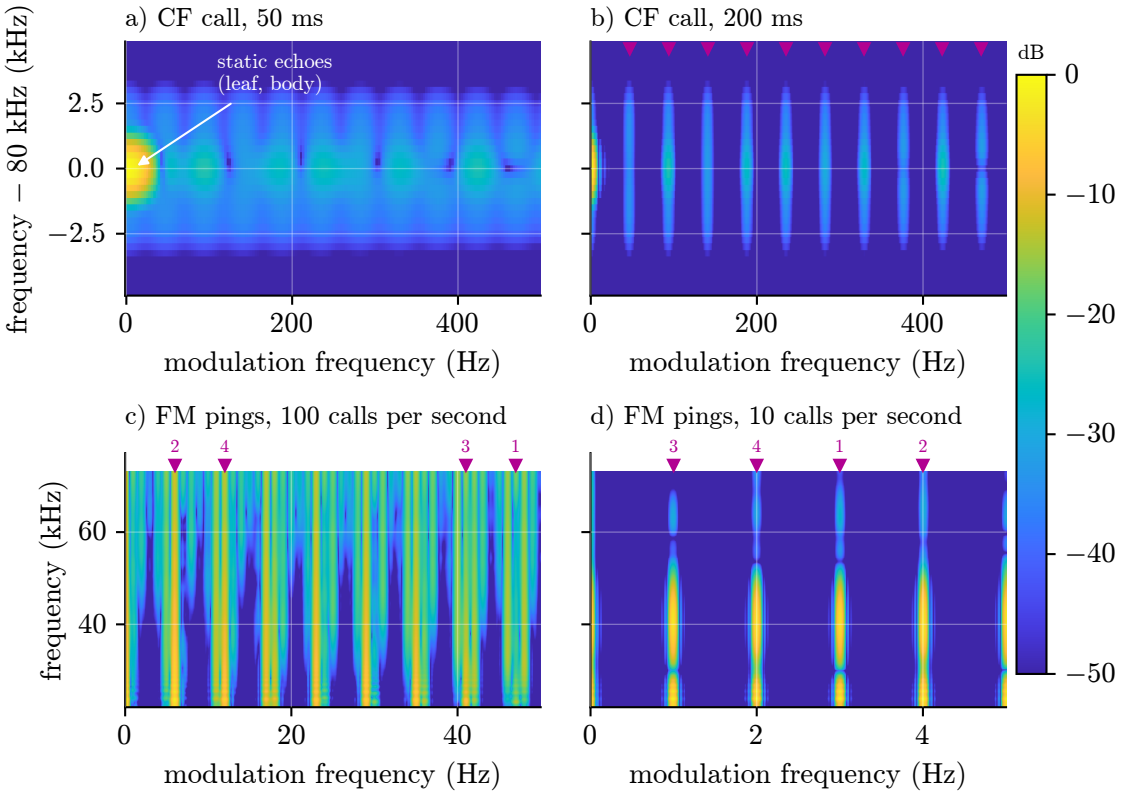


Figure 5: The JAMF of the fluttering insect, in dB relative to its maximum. a, b) The echo of a CF call at 80 kHz (STFT with 1 ms windows), with a static leaf 6 dB below a full glint, for a call of 50 and 200 ms. Static echoes stay at zero modulation frequency, and the wing adds lines at multiples of the wingbeat rate (triangles). c, d) Sequences of 256 FM pings at 100 and 10 calls per second (echo spectra divided by the call spectrum, as in Fig. 4). The modulation axis ends at half the call rate, and the triangles mark where the first four wingbeat harmonics fold to.

Key idea 3

The JAMF Fourier-transforms the power in every frequency bin of the spectrogram over time. Static reflectors stay at modulation frequency zero, and a fluttering wing appears at the wingbeat rate and its harmonics, at the acoustic frequencies of its glints. For a pulsed sonar, the modulation axis is sampled at the call rate and ends at half of it, and faster modulations fold back.

8 Detecting Flutter With a Robot Sonar

So what can a robot sonar with short FM calls do? It can locate the insect, as it would any other reflector, and it can notice that its echo is far less steady than the echoes of the static scene.

A simple detector tracks every reflector over several calls and computes the spread of its echo level. For a static reflector, the spread is set by the noise of the measurement, here assumed to be 1 dB. For our moth, the level jumps between the body echo and the glints, and its standard deviation over many calls is 4.6 dB (Fig. 3c). A threshold of a few dB separates the two well, without any knowledge of the insect. This spread is a one-number summary of the JAMF: it measures the power of the level fluctuations, summed over all nonzero modulation frequencies. Moving or rotating things (a fan, a waving hand, a branch in the wind) also flicker, so a flutter detector is a detector of fast motion rather than of insects specifically.

Estimating the wingbeat rate is harder, because the JAMF of a pulsed sonar shows it only at its apparent rate f_a, known up to multiples of the call rate. Randomizing the intervals between the calls breaks this ambiguity, since no single fold-back rate then fits all the samples, just as irregularly spaced microphones break the grating lobes of an array (radar calls this staggered pulse repetition frequencies). Following a single wingbeat takes either a long CF call, as the horseshoe bat uses, or a sonar that calls fast and a camera that shows what the wing was doing. This is the idea behind FL-RTIS, a sensor of our lab that combines a 3D sonar calling at a high rate with a high-speed camera on a shared clock, to record the echoes of insects in the field [4].

Exercise

1. How does the glint window change between 20 and 80 kHz? What does that mean for the shape of a glint in the spectrum of a broadband call?
2. A horseshoe bat flies at 5 m/s towards a hovering moth. By how much must it lower its 80 kHz call so that the echo of the body returns at 80 kHz?
3. Why do leaves moving in the wind not produce the same signature as a fluttering wing?
4. Show that $\Delta f_{\max} \cdot \tau \approx 2$ follows from the equations for the tip speed and the glint duration. What does this have to do with the time-bandwidth product of a signal?
5. A fan with three blades turns 20 times per second in front of a sonar that calls 30 times per second. At which modulation frequency does the fan appear in the JAMF of the pings, and could it be mistaken for clutter? What changes at 40 and at 50 calls per second?
6. A CF bat wants to separate the first two wingbeat harmonics of a moth beating its wings 47 times per second in the JAMF of a single call. About how long must the call be? Which STFT window would let it see modulations up to 1 kHz, and what does that window cost in acoustic frequency resolution?

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